Math 120A: Homework 7

Due: November 21, 2014

1. Read sections 5.1-3 and 5.6 in Pressley.

2. Do problems 4.5.1, 4.5.3, 5.1.2, 5.1.3, 5.2.2*, 5.2.3, and 5.3.3 in Pressley.

3. *Additional context for derivatives.* The notion of derivative introduced in class is a generalization of derivatives of maps $f : \mathbb{R}^n \to \mathbb{R}^m$. We say that f is differentiable at $\mathbf{p}$ if there is a linear map $T_{\mathbf{p}} : \mathbb{R}^n \to \mathbb{R}^m$ with the property that

$$f(\mathbf{p} + \mathbf{v}) = f(\mathbf{p}) + T_{\mathbf{p}}(\mathbf{v}) + ||\mathbf{v}||\epsilon(\mathbf{v})$$

where $\epsilon(\mathbf{v})$ is a function such that $\epsilon(\mathbf{v}) \to 0$ as $\mathbf{v} \to 0$.

   - Show that the ordinary derivative of functions $f : \mathbb{R} \to \mathbb{R}$ satisfies this definition.
   - Show that if f is differentiable at $\mathbf{p}$, then the matrix of $T_{\mathbf{p}}$ is the Jacobian of f.
   - The familiar theorem that differentiability at a point implies continuity at that point remains true for higher-dimensional derivatives. Use this and the last problem of Homework 1 to argue that the existence of the Jacobian at $\mathbf{p}$ does not imply differentiability at $\mathbf{p}$.

*Algebra background for 5.2.2: In class we mentioned the diagonalization theorem, which (inter alia) implies that if A is a symmetric $n \times n$ matrix, then there is an orthogonal matrix P such that PAP^t is diagonal. In fact the proof provides more information about P and PAP^t. Let $F(\mathbf{v}) = A\mathbf{v}$ be the linear transformation whose matrix with respect to the standard basis for $\mathbb{R}^n$ is A. One shows that there is an orthonormal basis $\mathcal{B} = \{v_1, v_2, \cdots, v_n\}$ for $\mathbb{R}^n$ consisting of eigenvectors of F. Then if the columns of P^t be the elements of this basis, the matrix PAP^t is the matrix of F with respect to $\mathcal{B}$. In particular PAP^t is diagonal. Exercise: What must the diagonal entries of PAP^t be?